### *Image result for bent coin image***Introduction – You can load a die but you can’t bias a coin[[1]](#footnote-2)**

*“The biased coin is the unicorn of probability theory—everybody has heard of it, but it has never been spotted in the flesh. As with the unicorn, you probably have some idea of what the biased coin looks like.”*

* Gelman and Nolan (2002).

In this activity, you’ll be given a half-dollar and decide whether or not it is *biased* in favor of landing on heads or tails and generate evidence to support your decision.

Our research question is:

*Is there evidence to suggest that your coin is biased?*

1. What numeric value (or parameter) are we estimating? Use appropriate notation.

\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. Label two competing claims:
2. Briefly describe the characteristics of your coin. Before collecting any data, do you think the chance that it will land on heads when tossed is 0.5 (i.e., do you think the null hypothesis is true and that the coin is *unbiased*)? Explain. If time is available, inspect and compare the coins of neighboring students.

### **Task 1 – Gathering Data**

Let’s gather some evidence to make an informal decision regarding a research question.

1. Flip your coin 30 times and record the outcome. For a coin flip to be considered valid, the coin must be flipped straight up in the air, so that it spins rapidly, and caught mid-air in the palm of your hand. The coin cannot be allowed to bounce.

|  |  |  |
| --- | --- | --- |
| Heads | Tails | Total |
|  |  |  |

1. Compute the sample proportion of heads,  . Given your findings, would you conclude that your coin is biased [i.e., reject in favor of ]?

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. What proportion of the students in your class collected data that led them to reject , the null hypothesis that their coin was unbiased?

### **Task 2 – Evaluating evidence: Randomization-based hypothesis tests**

In Task 1, we collected sample data at random and used it to make a (somewhat binary) decision regarding our research question. In Task 2, we’ll repeat this process in a more formal manner by applying a hypothesis test. You may have observed a sample proportion   that led you to believe your coin was biased.

However, we cannot yet be sure if this difference in the results we’d expect from an unbiased coin and the results we observed in the data represents a real disparity between 0.5 and , the true proportion of times your coin will land on heads, or a difference that is due to a chance effect associated with the random sampling process.

Generally, there is a little bit of fluctuation in sample data and we wouldn’t expect the sample proportion to be exactly 0.5, even if that was the true value of the population parameter.

The key to deciding in a more formal manner whether to believe the null hypothesis or the alternative hypothesis is to **figure out how much variability in   can be expected in a run of 30 coin flips.**

We can use technology to simulate the sampling distribution of   under the assumption that .

Navigate to the website: <https://shiny.stt.msu.edu/fairbour/OneProportion/>

1. Ensure that the Probability of Success is set of 0.5 and that the sample size is the same as your original experiment, .
2. Where the app says “Shuffle how many times?”, enter 5,000. Press the Shuffle button below. Continue to press the Shuffle button to increase the number of simulations. As the number of simulations increases, what proportion of the simulated coin-flip samples produce a   value as extreme as the one in your original sample?

Recall our two competing claims:

**the coin is unbiased.** The true proportion of flips that land on heads is and any observed difference in sample results is due to chance.

**the coin is biased.** The true proportion of flips that land on heads is something other than 0.5 [i.e., ] and the observed difference in sample results reflects the fact that the null value poorly describes the behavior of the biased coin.

When we conduct formal studies, we reject a hypothesis if our data strongly conflict with it. In this instance, we simulated how the potentially-biased coin ought to behave if it were, in fact, unbiased, and then computed the proportion of simulated   values that deviated from the expected success rate of as much or more than our observed  .

1. Given your answer in (8), do you believe your coin is biased? How does your decision now compare to your earlier decision in (5)?

**Task 3 – Try another example**

In the previous task, you likely learned that bending a coin *does not* in fact bias its chance of landing on heads. Is this also the case for coins that are **spun**? We establish two competing claims:

**the coin is unbiased.** The true proportion of **spins** that end on heads is and any observed difference in sample results is due to chance.

**the coin is biased.** The true proportion of **spins** that end on heads is something other than 0.5 [i.e., ] and the observed difference in sample results reflects the fact that the null value poorly describes the behavior of the biased coin.

1. **Spin** your coin 30 times and record the outcome. The coin must end on the same surface it began spinning on [i.e., do not allow it to drop from the table to the floor].

|  |  |  |
| --- | --- | --- |
| Heads | Tails | Total |
|  |  |  |

1. Compute the sample proportion of heads,  . Given your findings, would you conclude that your coin is biased [i.e., reject in favor of ]?

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Return to the website: <https://shiny.stt.msu.edu/fairbour/OneProportion/>

1. Ensure that the Probability of Success is set of 0.5 and that the sample size is the same as your original experiment, .
2. Where the app says “Shuffle how many times?”, enter 5,000. Press the Shuffle button below. Continue to press the Shuffle button to increase the number of simulations. As the number of simulations increases, what proportion of the simulated coin-flip samples produce a   value as extreme as the one in your original sample?

Recall our two hypotheses:

**the coin is unbiased.** The true proportion of **spins** that end on heads is and any observed difference in sample results is due to chance.

**the coin is biased.** The true proportion of **spins** that end on heads is something other than 0.5 [i.e., ] and the observed difference in sample results reflects the fact that the null value poorly describes the behavior of the biased coin.

1. Based on your result, which interpretation would you choose – the null hypothesis or the alternative? State which and briefly explain your answer.

1. This activity is inspired by an article by Andrew Gelman and Deborah Nolan by the same name. [↑](#footnote-ref-2)